

PROBLEM SET 8

1) $\Delta m v_e = m \Delta v$ Constant exhaust velocity

By separation of variables,

$$\int_{m_i}^{m_f} \frac{1}{m} dm = \frac{1}{v_e} \int_{v_i=0 \text{ ms}^{-1}}^{v_f} dv$$

, where m_i = initial mass of rocket (kg)

m_f = final mass of rocket (kg)

v_i = initial velocity of rocket = 0 ms^{-1}

v_f = final velocity of rocket

v_e = Exhaust velocity (ms^{-1})

$$\ln |m_f/m_i| = \frac{1}{v_e} \cdot v_f$$

$$e^{v_f/v_e} = \frac{m_f}{m_i}$$

$$\therefore m_f = m_i e^{v_f/v_e}$$

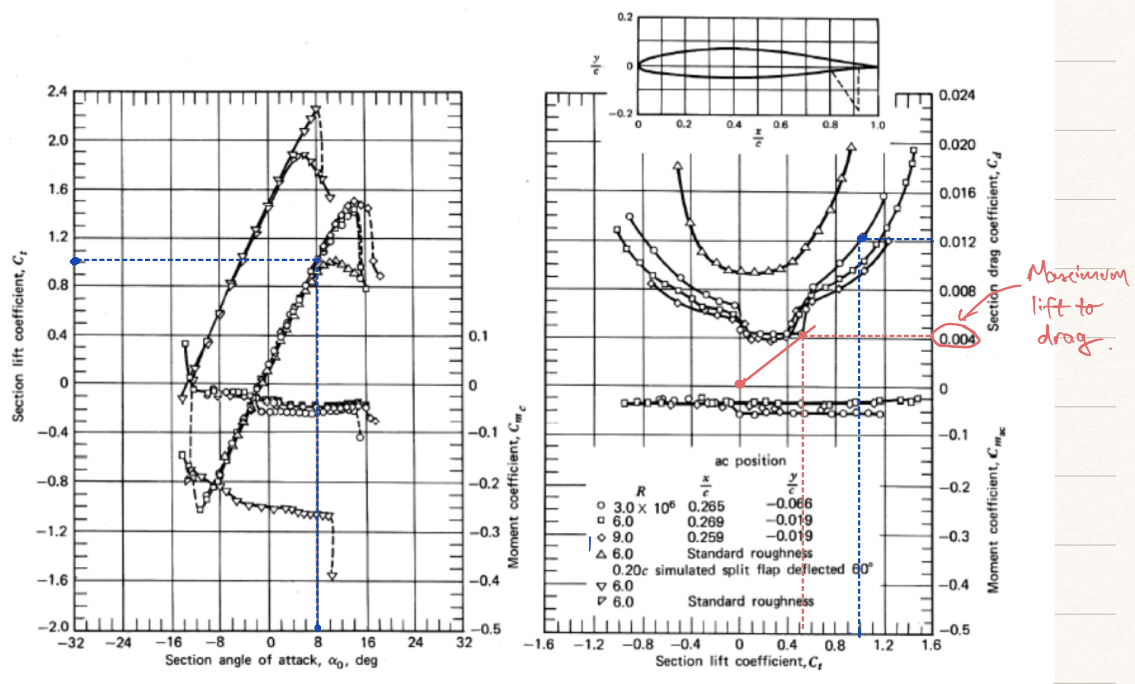
$$v_e = 5500 \text{ ms}^{-1}, \quad v_f = 12000 \text{ ms}^{-1}$$

$$\therefore m_f = m_i e^{\frac{12000}{-5500}} \quad \leftarrow \text{(-)ve because thrust is produced opposite to direction of motion.}$$

$$m_f = 0.112836 m_i$$

$$\therefore \% \text{ of mass burnt (mass of fuel)} = (1 - 0.112836) \times 100$$
$$= 88.72\%$$

2)



$$R = \frac{\rho u L}{\mu}, \quad \rho = \text{Density of air} = \frac{P}{RT} = \frac{100.5 \times 1000}{287.05 \times 303}$$

$$\rho = 1.155 \text{ kg/m}^3$$

$$u = \text{Flow stream velocity} = 50 \text{ m/s}$$

$$L = \text{Chord length} = 0.7 \text{ m.}$$

$$\mu = \text{Viscosity of air} = 1.458 \times 10^{-6} \times \frac{T^{3/2}}{T+110.4}$$

$$\text{Hence, } R = \frac{1.55 \times 50 \times 0.7}{1.86 \times 10^{-5}}$$

$$\text{where } T = 303 \text{ K}$$

$$\therefore \mu = 1.86 \times 10^{-5} \text{ N/m}^2 \text{ s}$$

$$= 2.17 \times 10^6 \approx 3 \times 10^6$$

$$\text{Therefore, } C_l = 1, \quad C_D = 0.012 \Rightarrow L/D \text{ ratio} = 83.33$$

$$\text{Max } L/D = 0.5/0.004 = 125$$

Therefore at this angle of attack, L/D is 66.67%
of the maximum L/D ratio.

$$3) \text{ a. } C_L = 0 \Rightarrow 0 = 2\pi\alpha + 0.4$$

$$\text{therefore, } \alpha = -0.4/2\pi = -0.06 \text{ rad} \\ = -3.65^\circ$$

$$\text{b. } \frac{dC_L}{d\alpha} = \frac{\Delta C_L}{\Delta\alpha} = \frac{C_L - 0}{\alpha - (-0.06)} = \frac{2\pi}{1 + \frac{2}{6}}$$

$$\frac{1}{\alpha + 0.06} = \frac{2\pi}{1 + \frac{2}{6}} \Rightarrow \alpha = 0.152 \text{ rad} = 8.7208^\circ$$

$$\text{c. } \alpha_i = \frac{C_L}{\pi A} = \frac{1}{\pi \times 6} = 0.053 \text{ rad} = 3.04^\circ$$

$$\text{d. } C_{Di} = \frac{C_L^2}{\pi A} = \frac{1^2}{\pi \times 6} = 0.053$$